Influence of long range interactions and disorder correlations on the critical behavior of the Ising model

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A work done with G. Tarjus (LPTMC), M. Tissier (LPTMC) and Y. Sakamoto (College of Science and Technology, Japan)
Pair interaction potential with a power law decay:

\[ V(r) \sim \frac{1}{r^{d+\sigma}} \quad (r \gg 1) \]

Moderately long range interactions: fast decay \( \sigma > 0 \).

⇒ Use the usual equilibrium statistical physics to study phase transitions and critical phenomena.
• Applications in condensed matter physics.
• LRI modify the critical behavior and tend to supress fluctuations.
  ⇒ New universality classes depending continuously on $\sigma$.
  ⇒ Phase transitions even in one dimensional systems (decrease of the upper and the lower critical dimensions).
• Use in disordered systems theory (spin glasses, Random Field Ising Model):
  Vary $d$ in the SR model $\simeq$ Vary $\sigma$ with $d$ fixed in the LR model.
  ⇒ Work in one dimensional systems (simulations, real space RG).
Field theory: the action

\( \phi^4 \) field theory for the Ising Model with long range interactions:

\[
\beta \mathcal{H}[\phi] = S[\phi] = \int d^d x \, d^d x' \, j(|x - x'|) \phi(x) \phi(x') + \int d^d x \, \frac{g}{4!} \phi(x)^4
\]

\[
\sim \int \frac{d^d q}{(2\pi)^d} \frac{1}{2} \phi(q) (cq^2 + c'(q^2)^{\frac{\sigma}{2}} + \tau) \phi(-q) + \int d^d x \, \frac{g}{4!} \phi(x)^4
\]

\[
= \int d^d x \, \frac{1}{2} \phi(x) (-c \partial^2 - c'(\partial^2)^{\frac{\sigma}{2}} + \tau) \phi(x) + \frac{g}{4!} \phi(x)^4
\]

where \( j(r) \sim r^{-d-\sigma} \) and \( (\partial^2)^{\alpha} \phi(x) = -\int \frac{d^d q}{(2\pi)^d} (q^2)^{\alpha} \tilde{\phi}(q) e^{iq \cdot x} \)

the fractional laplacian.

\( \Rightarrow \) Non analytical 2 points function in momentum dependence.

\( \Rightarrow \) \( \sigma \) must be compared to 2:

- \( \sigma < 2 \): Long distance physics dominated by the LR term \( q^\sigma \).
- \( \sigma \geq 2 \): the SR critical behavior is recovered.
Field theory : results about the critical behavior

• MF theory ⇒ \( \sigma \)-dependent exponents in the LR regime:

\[
\eta = 2 - \sigma \quad \nu = \sigma^{-1} \quad \text{for } \sigma < 2
\]

\[
(\eta = 0 \quad \nu = 1/2 \quad \text{for } \sigma \geq 2)
\]

• MF theory applies when \( \sigma \leq \frac{d}{2} \) i.e. \( d_{\text{up}} = 2\sigma \) for \( \sigma < 2 \)

\( (d_{\text{up}} = 4 \) for \( \sigma \geq 2).\)

• Perturbative RG at 2 loops level in the LR regime \( \epsilon = 2\sigma - d \).

uestioned perturbative RG results:

• No renormalization of the non analytic term \( q^{\sigma} \)

\( \Rightarrow \eta = 2 - \sigma \) always applies in the LR regime, even out of the classical behavior (RG vs simulations).

• Limit between the two regimes in \( \sigma = 2 - \eta_{SR} \) instead of \( \sigma = 2 \) (RG vs RG & RG vs simulations).
Field theory: phase diagram

If:

- No renormalization of the $q^\sigma$ term.
- Transition between the 2 regimes in $\sigma = 2 - \eta_{SR}$.

$\Rightarrow$ The exponents are continuous functions of $\sigma$. 

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Non Perturbative Renormalization Group

Use the Wetterich flow equation for the effective average action $\Gamma_k$.

Derivative expansion with a non analytical term:

$$\Gamma_k[\varphi] = \int d^d x \left\{ \frac{1}{2} Z_k(\varphi(x))(\partial_\mu \varphi(x))^2 - \frac{1}{2} Y_k \varphi(x)(\partial^2)\frac{\sigma}{2} \varphi(x) + V_k(\varphi(x)) \right\}$$

- One finds $\partial_t Y_k = \frac{d}{dq} \partial_t \tilde{\Gamma}_k^{(2)}(q, -q)|_{q=0} = 0$
  $\Rightarrow$ One sets $Y_k = 1$

- Flow equations for $V_k(\varphi)$ and $Z_k(\varphi)$ with the non analytic propagator:

  $$\tilde{G}_k[\varphi](q^2) = \frac{1}{q^\sigma + Z_k(\varphi)q^2 + V''_k(\varphi) + R_k(q^2)}$$

- Rescaling to find the fixed point in the LR regime:

  $$\tilde{\varphi} = k^{\frac{\sigma-d}{2}} \varphi \quad \tilde{Z}_k(\tilde{\varphi}) = k^{2-\sigma} Z_k(\varphi) \quad \tilde{V}_k(\tilde{\varphi}) = k^{-d} V_k(\varphi)$$

  fixed anomalous dimension $\eta_k = 2 - \sigma$. 
Results of NPRG

- No renormalization of the non analytical term
  \[ \Rightarrow \eta = 2 - \sigma \] in the LR regime.

- One Recovers the stable gaussian fixed point for \( \sigma \leq \frac{d}{2} \) and the perturbative one loop result in \( \epsilon = 2\sigma - d \).

- Numerical integration of the flow equations \( \partial_t \tilde{V}_k(\tilde{\varphi}) \) and \( \partial_t \tilde{Z}_k(\tilde{\varphi}) \) to find the fixed point in the non classical LR regime and check the limit between the LR and SR behaviors.
Numerical results in 2 dimensions

⇒ Consistent results with the change of regime in $\sigma = 2 - \eta_{SR}$ and not $\sigma = 2$

$\eta_{SR} \approx 0.28$ using NPRG

$\eta_{SR}^{\text{exact}} = 0.25$
Long range disorder correlations

Random Field Ising Model: Ising Model in presence of a quenched random magnetic field \( h(x) \).

**SR version of RFIM:** \( \overline{h(x)h(y)} = \Delta \delta(x - y) \)

- SUSY in the associated field theory at \( T = 0 \).
  - Dimensional Reduction property \( d \mapsto d - 2 \):
    - critical behavior of the disordered model in \( d \) dimensions
    = critical behavior of the pure model in dimension \( d - 2 \)
- SUSY and DR break below a critical dimension \(~ 5.15\) (shown by NP-F RG).

**LR version of RFIM:** \( \overline{h(x)h(y)} \sim \Delta |x - y|^{\rho - d} \)

- RFIM with LR interactions (\( \sigma \)) and LR disorder correlations (\( \rho \)).
- Same SUSY as in the SR version when \( \rho = 2 - \sigma \).
- Recheck the validity of DR in the supersymmetric LR version of RFIM as function of \( \sigma \).
Long range disorder correlations

Work in progress.

Thank for your attention.

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