A non-perturbative, real-space, Renormalization Group approach to the Glass Transition.

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Glassy systems in nature and science

Glasses, granular systems, plastic materials, K-sat problem....

Search for a universal behavior: is there a glass transition?

Focus on supercooled liquids and glass formation.
Glassy systems in nature and science

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Focus on supercooled liquids and glass formation.

Impressive growth of relaxation time.

Is the dynamical slowing down due to activated relaxation?

\[ \tau = \tau_0 \exp\left(\frac{A}{(T - T_0)}\right) \]

\[ \tau = \tau_0 \exp\left(\frac{\Delta(T)}{T}\right) \]

\[ \Delta(T) = AT/(T - T_0) \]

II. JUSTIFICATION OF A POTENTIAL ENERGY BARRIER DESCRIPTION

I will begin by an attempt to justify my belief that, when all is said and done, the existence of potential energy barriers large compared to thermal energy are intrinsic to the occurrence of the glassy state, and dominate flow, at least at low temperatures. This is in agreement with the entropy concepts of Gibbs, but not with the free volume viewpoint.

Goldstein (1969)
Glassy systems in nature and science

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Focus on supercooled liquids and glass formation.

Impressive growth of relaxation time.

\[ \tau = \tau_0 \exp\left(\frac{A}{(T - T_0)}\right) \]

Is the dynamical slowing down due to activated relaxation?

Lack of a standard correlation length-scale.

\[ \xi = ? \]
A tentative Mean-Field approach

a) a low temperature multi-states phase space
b) a new kind of thermodynamic transition

\[ s_c = \log(\mathcal{N})/N \]

A new order parameter: the probability distribution of the overlap \( q \), i.e. the similarity between two typical configurations.
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A tentative Mean-Field approach

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\[ V(q_{ab}) = \sum_{ab} \left( \frac{t}{2} q_{ab}^2 - \frac{u+w}{3} q_{ab}^3 + \frac{y}{4} q_{ab}^4 \right) - \frac{u}{3} \sum_{abc} q_{ab} q_{bc} q_{ca} \]

\[ s_c = \log(\mathcal{N}) / N \]

A new order parameter: the probability distribution of the overlap \( q \), i.e. the similarity between two typical configurations.
A tentative Mean-Field approach and beyond...

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Need to add non-perturbative fluctuations by a Renormalization Group approach
Ginzburg-Landau theory for replicated liquids

\[ V(q_{ab}) = \sum_{ab}^{m} \left( \frac{t}{2} q_{ab}^2 - \frac{u + w}{3} q_{ab}^3 + \frac{y}{4} q_{ab}^4 \right) - \frac{u}{3} \sum_{abc}^{m} q_{ab} q_{bc} q_{ca} \]

The Migdal-Kadanoff Renormalization group

\[ W_{n}(q_{1}^{ab}, q_{2}^{ab}) = -2^{d-1} \log \int \Pi_{ab} d q_{ab} \exp(-W_{n-1}(q_{1}^{ab}, q^{ab}) - W_{n-1}(q^{ab}, q_{2}^{ab}) - V(q^{ab})) \]

\[ W_{n}(q_{1}, q_{2}) = 2^{d-1} \min_{q} \{ W_{n-1}(q_{1}, q) + W_{n-1}(q, q_{2}) + V(q) \} \]
Ginzburg-Landau theory for replicated liquids

$$V(q_{ab}) = \sum_{ab}^{m} \left( \frac{t}{2} q_{ab}^2 - \frac{u + w}{3} q_{ab}^3 + \frac{y}{4} q_{ab}^4 \right) - \frac{u}{3} \sum_{abc} q_{ab} q_{bc} q_{ca}$$

$$S(Q_{ab}) = \int \frac{d^d x}{a_0^d} \left[ \frac{a_0}{2} \sum_{ab} (\partial q_{ab})^2 + V(q_{ab}) \right]$$

The Migdal-Kadanoff Renormalization group

$$W_n(q_{1}^{ab}, q_{2}^{ab}) = -2^{d-1} \log \int \prod_{ab} dq_{ab} \exp(-W_{n-1}(q_{1}^{ab}, q_{ab}) - W_{n-1}(q_{ab}, q_{2}^{ab}) - V(q_{ab}))$$

$$W_n(q_1, q_2) = 2^{d-1} \min_{q} \{W_{n-1}(q_1, q) + W_{n-1}(q, q_2) + V(q)\}$$
The Renormalization Group flow

\[ W_n(q_1, q_2) = 2^{d-1} \min_q \{ W_{n-1}(q_1, q) + W_{n-1}(q, q_2) + V(q) \} \]

At the cooperative-static length scale the renormalized liquid is a simple liquid

\[ l_s \sim (T - T_K)^{-1} \]

The renormalized system

\[ W_n(q_1, q_2) = 2^{d-1} \min_q \{ W_{n-1}(q_1, q) + W_{n-1}(q, q_2) + V(q) \} \]

• At the cooperative-static length scale the renormalized liquid is a simple liquid

\[ l_s \sim (T - T_K)^{-1} \]

• Two regimes of the flow: \( l < l_s \) \( W(q^*, 0) \gg W(q^*, q^*) \)

\( l > l_s \) \( W(q^*, 0) < W(q^*, q^*) \) \( W(q^*, 0) \gg T \)

\[ \ln(\tau) \propto \frac{W(q^*, 0)}{T} \propto \frac{1}{(T - T_K)^{d-1}} \]
Within and beyond the present approximation

Within this approximation (saddle point):

• Critical behavior of a first order discontinuity fixed point

Fluctuations beyond the saddle point are crucial!

• Transition may disappear
• Critical point can change nature
• A new fixed point may appear

Beyond the saddle point (i.e. including the disorder):

• Adding fluctuations to the present M-K field theory approach
• Functional Renormalization Group approach

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Thank you for your attention!